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Properties of $X(3872)$ as a hadronic molecule with a negative parity

Masayasu Harada

*Department of Physics, Nagoya University, Nagoya, 464-8602, Japan.
 harada@hken.phys.nagoya-u.ac.jp*

Yong-Liang Ma

*Department of Physics, Nagoya University, Nagoya, 464-8602, Japan.
 yлма@hken.phys.nagoya-u.ac.jp*

We discuss the possible interpretation of $X(3872)$ as a DD^* hadronic molecule with $J^{PC} = 2^{-+}$. Using the phenomenological Lagrangian approach, we studied its radiative and strong decay properties. We find that our model with about 97.6% isospin zero component explains the existing data nicely. We predict the partial widths of the radiative and strong decays of $X(3872)$ and show that the measurement of the ratio $\mathcal{B}(X(3872) \rightarrow \chi_{c0}\pi^0)/\mathcal{B}(X(3872) \rightarrow \chi_{c1}\pi^0)$ may signal the nature of $X(3872)$.

Keywords: Exotic state; hadronic molecule; negative parity.

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1. Introduction

In our recent work ¹, we presented a composite model for $X(3872)$ with $J^{PC} = 2^{-+}$ which is favored by recent experiment ² by regarding it as a DD^* bound state. Based on an effective Lagrangian approach, we fitted the parameters using the existing data and found that $X(3872)$ is dominantly an isospin singlet. With this clear and straight model, we calculated the radiative and strong decay properties of $X(3872)$.

2. Hadronic Molecular Structure of $X(3872)$ with $J^{PC} = 2^{-+}$

We write the wave function of $X(3872)$ in terms of charge eigenstates as

$$|X(3872)\rangle = \frac{\cos\theta}{\sqrt{2}}|D^0\bar{D}^{*0}\rangle + \frac{\sin\theta}{\sqrt{2}}|D^+D^{*-}\rangle + \text{C.c.}, \quad (1)$$

or equivalently, in terms of the isospin eigenstates as

$$|X(3872)\rangle = \cos\phi|X(3872)\rangle_{I=0} + \sin\phi|X(3872)\rangle_{I=1}, \quad (2)$$

with $\cos\theta = (\cos\phi + \sin\phi)/\sqrt{2}$ and $\sin\theta = (\cos\phi - \sin\phi)/\sqrt{2}$ and

$$|X(3872)\rangle_{I=0,1} = \frac{1}{2}\left(|D^0\bar{D}^{*0}\rangle \pm |D^+D^{*-}\rangle\right) + \text{C.c.}, \quad (3)$$

Our composite model is based on the effective Lagrangian describing the interaction between $X(3872)$ and its constituents

$$\begin{aligned} \mathcal{L}_X = & \frac{i}{\sqrt{2}} X^{\mu\nu}(x) \int dx_1 dx_2 \Phi_X((x_1 - x_2)^2) \delta(x - \omega_v x_1 - \omega_p x_2) \\ & \times \left\{ g_X^N \left[C_{\mu\nu}^N(x_1, x_2) + C_{\nu\mu}^N(x_1, x_2) - \frac{1}{4} g_{\mu\nu} C_{\alpha}^{N;\alpha}(x_1, x_2) \right] \right. \\ & \left. + g_X^C \left[C_{\mu\nu}^C(x_1, x_2) + C_{\nu\mu}^C(x_1, x_2) - \frac{1}{4} g_{\mu\nu} C_{\alpha}^{C;\alpha}(x_1, x_2) \right] \right\}, \end{aligned} \quad (4)$$

where $g_X^N(g_X^C)$ is the effective coupling constant for the interaction between $X(3872)$ and its neutral (charged) constituents. ω_v and ω_p are mass ratios with definitions $\omega_v = m_{D^*}/(m_{D^*} + m_D)$, $\omega_p = m_D/(m_{D^*} + m_D)$ with $m_D(m_{D^*})$ as the mass of the constituent $D(D^*)$ meson. The function $\Phi_X((x_1 - x_2)^2)$ illustrates the finite size of the molecule, and in the calculation, its Fourier transform $\tilde{\Phi}_X(p^2) = \exp(p^2/\Lambda_X^2)$ with the size parameter Λ_X parameterizing the distribution of the constituents inside the molecule has been applied. The tensor $C_{\mu\nu}^N$ is defined as

$$C_{\mu\nu}^N(x_1, x_2) = \bar{D}_\mu^{*0}(x_1) \partial_\nu D^0(x_2) + D_\nu^{*0}(x_1) \partial_\mu \bar{D}^0(x_2). \quad (5)$$

Substituting the neutral constituents with the corresponding charged ones, one can get the explicit form of $C_{\mu\nu}^C$.

Relation between the mixing angle θ defined in Eq. (1) and the coupling constant $g_X^N(g_X^C)$ can be yielded using the compositeness condition $Z_X = 0$ ^{3,4} with Z_X as the wave function renormalization constant of $X(3872)$ which is defined as

$$Z_X = 1 - g_X^2 \frac{d}{dp^2} \Sigma_X(p^2) \Big|_{p^2=m_X^2}, \quad (6)$$

where $g_X^2 \Sigma_X(p^2)$ relates to the mass operator via the relation

$$\Pi_X^{\mu\nu;\alpha\beta}(p^2) = \frac{1}{2} (g^{\mu\alpha} g^{\nu\beta} + g^{\mu\beta} g^{\nu\alpha}) g_X^2 \Sigma_X(p^2) + \dots, \quad (7)$$

with “...” denoting terms do not contribute to the mass renormalization of $X(3872)$. Explicitly, we have the following relations

$$\cos \theta = g_X^N \frac{d}{dp^2} \Sigma_X^N(p^2) \Big|_{p^2=m_X^2}, \quad \sin \theta = g_X^C \frac{d}{dp^2} \Sigma_X^C(p^2) \Big|_{p^2=m_X^2}, \quad (8)$$

where $\Sigma_X^N(\Sigma_X^C)$ corresponds to the case with neutral (charged) constituents.

Concerning the uncertainty of the $X(3872)$ mass, we express $m_X = m_{D^*0} + m_{D^0} - \Delta E$ with $\Delta E > 0$ as the binding energy and $m_{D^*0} = 2006.97$ MeV and $m_{D^0} = 1864.84$ MeV⁵. In the computation, we take $\Delta E = 0.5, 1.0$ and 1.5 MeV.

3. Radiative and Strong Decay Properties

Using the model (4), we calculated the radiative and strong decay widths. Except the coupling constant between the molecule and its constituents which is determined

using the compositeness condition, other coupling constants are borrowed from the effective Lagrangian approaches.

In our numerical calculation, we take the mixing angle ϕ and the size parameter Λ_X as free parameters to fit the following data

$$R_1 = \frac{\mathcal{B}(X \rightarrow \gamma\psi(2S))}{\mathcal{B}(X \rightarrow \gamma J/\psi)} = 3.4 \pm 1.4 \text{ BaBar } ^7, \quad (9)$$

$$R_2 = \frac{\mathcal{B}(X(3872) \rightarrow J/\psi\pi^+\pi^-\pi^0)}{\mathcal{B}(X(3872) \rightarrow J/\psi\pi^+\pi^-)} = 1.0 \pm 0.4(\text{stat.}) \pm 0.3(\text{syst.}) \text{ Belle } ^6, \quad (10)$$

$$R_3 = \frac{\mathcal{B}(X(3872) \rightarrow \gamma J/\psi)}{\mathcal{B}(X(3872) \rightarrow J/\psi\pi^+\pi^-)} = 0.14 \pm 0.05 \text{ Belle } ^6; 0.33 \pm 0.12 \text{ BaBar } ^7(11)$$

We find that we can reproduce all the above data except R_3 by Belle ⁶. If the Belle data for R_3 is favored in future experiments, our model will be excluded. It should be stressed that we can reproduce the above three data with only two parameters so our model is nontrivial. Combining with the data $R_4 = \mathcal{B}(X(3872) \rightarrow \gamma\psi(2S))/\mathcal{B}(X(3872) \rightarrow J/\psi\pi^+\pi^-) = 1.1 \pm 0.4$ from BaBar ⁷, we restrict ourselves to the case with $\phi = 9^\circ$ and the corresponding $\Lambda_X = 2.8 - 3.0$ GeV. Our results of relevant ratios are given in Table. 1.

Table 1. The ratio with fitted parameters $\phi = 9^\circ$ and $\Lambda_X = 2.8 - 3.0$ GeV. The last two rows are data.

$\Delta E(\text{MeV})$	R_1	R_2	R_3	R_4
0.5	2.430 – 3.486	0.964 – 1.035	0.369 – 0.283	0.897 – 0.987
1.0	2.433 – 3.504	0.917 – 0.971	0.363 – 0.274	0.883 – 0.960
1.5	2.433 – 3.516	0.874 – 0.914	0.358 – 0.266	0.871 – 0.935
	3.4 ± 1.4 ⁷	$1.0 \pm 0.4 \pm 0.3$ ⁶	0.14 ± 0.05 ⁶	1.1 ± 0.4 ⁷
			0.33 ± 0.12 ⁷	

Table 2. Effective coupling constants g_X^N and g_X^C from the fitted parameters ϕ and Λ_X .

$\Delta E(\text{MeV})$	g_X^N	g_X^C
0.5	16.29 – 15.84	12.64 – 12.40
1.0	16.38 – 15.94	12.68 – 12.44
1.5	16.46 – 16.04	12.72 – 12.48

Table 3. The partial widths related to the data from the fitted parameters ϕ and Λ_X .

ΔE (MeV)	$\Gamma(X \rightarrow \gamma J/\psi)$ (KeV)	$\Gamma(X \rightarrow \gamma\psi(2S))$ (KeV)	$\Gamma(X \rightarrow J/\psi\pi^+\pi^-\pi^0)$ (KeV)	$\Gamma(X \rightarrow J/\psi\pi^+\pi^-)$ (KeV)
0.5	2.085 – 1.872	5.066 – 6.525	5.447 – 6.842	5.648 – 6.612
1.0	2.082 – 1.864	5.065 – 6.533	5.253 – 6.606	5.730 – 6.802
1.5	2.079 – 1.859	5.058 – 6.537	5.067 – 6.380	5.801 – 6.977

Table 4. Partial widths for $X(3872) \rightarrow \chi_{cJ}\pi^0$ decays.

ΔE (MeV)	$\Gamma(X \rightarrow \chi_{c0}\pi^0)$ (KeV)	$\Gamma(X \rightarrow \chi_{c1}\pi^0)$ (KeV)	$\Gamma(X \rightarrow \chi_{c2}\pi^0)$ (KeV)
0.5	22.41 – 21.97	0.294 – 0.276	207.5 – 8.335
1.0	22.60 – 22.40	0.296 – 0.281	211.8 – 8.618
1.5	22.78 – 22.76	0.296 – 0.285	215.9 – 8.880

In Table. 2 we present our numerical results of the coupling constants g_x^N and g_x^C . Our bigger results than the corresponding ones in the case $X(3872)$ with $J^{PC} = 1^{++}$ ⁸ are because we need stronger attractive interaction in the P -wave case to compensate the repulsive interaction induced by angular momentum.

In terms of the isospin basis, one can write the wave function of $X(3872)$ as

$$|X(3872)\rangle = 0.988 \times |X(3872)\rangle_{I=0} + 0.156 \times |X(3872)\rangle_{I=1}. \quad (12)$$

But this dominant isospin singlet component does not mean the decay $X \rightarrow J/\psi\pi^+\pi^-$ is strongly suppressed compared to $X \rightarrow J/\psi\pi^+\pi^-\pi^0$ decay because the later process is strongly suppressed by the phase space factor. The explicit calculation yields the ratio consistent with the experimental data given by (10).

In Table. 3 we give the partial widths for decays with J/ψ or $\psi(2S)$ in the final states. All results are of order of KeV. The inclusion of other components may change our results. In case of $J/\psi\omega$ and $J/\psi\rho$ are the components^{8,9}, the magnitudes of the strong decay widths may be increased and the results depend on the probability of these components. In this sense, if the strong decay widths for tensor $X(3872)$ are observed bigger than our present results, one may conclude that it cannot be a pure DD^* molecule and other component should be included. If the $X(3872)$ is regarded as a mixing state of $c\bar{c}$ and DD^* , one may borrow the lesson from the $X(3872)$ with 1^{++} case¹⁰ to naively expect that this change of the wave function of $X(3872)$ may increase the magnitude of radiative decay width give in Table. 3. Although all these cases should be calculated in detail, the precise measurement of the strong decays can provide some clues on the structure of $X(3872)$.

Table. 4 is the summary of our numerical results for the decays of $X(3872) \rightarrow \chi_{cJ}\pi^0$. One can yield the following ratio of the partial widths

$$\Gamma(X \rightarrow \chi_{c0}\pi^0) : \Gamma(X \rightarrow \chi_{c1}\pi^0) \simeq 1 : 0.013, \quad (13)$$

which indicates that, compared with $X \rightarrow \chi_{c0}\pi^0$, $X \rightarrow \chi_{c1}\pi^0$ is strongly suppressed.

Similarly to the radiative decay, including other components may change our numerical results. In case of the $c\bar{c}$ and $J/\psi\omega$ components are included, the magnitudes for the partial widths might be changed but their ratio must be kept since they are definitely isospin singlets so they do not contribute to these decays. On the contrary, complication arises if $X(3872)$ has a $J/\psi\rho$ component. So that, in the case $X(3872)$ with $J^{PC} = 2^{-+}$, the strong suppression of the decay $X(3872) \rightarrow \chi_{c1}\pi^0$

compared with the decay $X(3872) \rightarrow \chi_{c0}\pi^0$ may signal the pure DD^* molecular structure of $X(3872)$.

4. Conclusion

By regarding the hidden charm meson $X(3872)$ with $J^{PC} = 2^{-+}$ as a DD^* bound state, we found that our model with dominant isospin zero component can explain the existing data quite well. We also predicted the partial widths for $X(3872) \rightarrow \gamma J/\psi$, $X(3872) \rightarrow \gamma\psi(2S)$, $X(3872) \rightarrow J/\psi\pi^+\pi^-$, $X(3872) \rightarrow J/\psi\pi^+\pi^-\pi^0$ and $X(3872) \rightarrow \chi_{cJ}\pi^0$. And, from our study, comparison of these values with the future experiments will shed a light on the nature of $X(3872)$.

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